Journal of International Economics 19 (1985) 161–170. North-Holland

A RESULT ON THE TRANSFER PROBLEM IN INTERNATIONAL TRADE THEORY

Mukul MAJUMDAR and Tapan MITRA*

Cornell University, Ithaca, NY 14853, USA

Received March 1983, revised version received March 1985

The purpose of this paper is to provide a result on the welfare effect of a transfer, in a general equilibrium exchange model, with no arbitrary restrictions on either the number of commodities traded or the number of countries. Our result can be stated as follows. The donor country is worse off after a transfer, if (a) all goods are 'net substitutes' for the donor; (b) all goods are 'gross substitutes' for the other countries; and (c) all goods are 'normal' for all countries.

1. Introduction

The 'transfer problem' in international trade theory can be roughly described as follows (in the context of a general equilibrium exchange model with m countries and n goods). Suppose one country (the 'donor') gives away ('transfers') some of its endowment to another country (a 'recipient') or to some or all of the other countries,

(a) how will the post-trasnfer equilibrium price compare to the pre-transfer one?

(b) how will the post-trasnfer equilibrium welfares of the countries compare to the pre-transfer levels?

The effect of a transfer studied in question (a) is called a 'positive effect'; that in question (b) a 'welfare effect'.

Both the above questions were posed by Samuelson (1952, 1954). Since then, a considerable body of literature has developed on the transfer problem. Our interest is in the question on the 'welfare effect', and it is useful to summarize the findings in the literature on this question.

Most of the literature focuses on the two-country, two-commodity model. The main proposition that emerges from this model is that if the transfer is

*We became interested in this problem upon examining some unpublished manuscripts (reporting the research of Bhagwati, Brecher, Hatta) kindly sent to us by Professor Bhagwati. We are indebted to Avinash Dixit, Anjan Mukherji and Henry Wan for some very useful discussions. The present version has benefited greatly from comments by a referee and the editor. The research of both authors was supported by a National Science Foundation Grant. The research of the second author was, in addition, supported by an Alfred P. Sloan Research Fellowship.

'small', the donor country cannot be better off (and so the recipient country cannot be worse off) after the transfer, provided the Walrasian (World) equilibrium is 'locally' stable under the usual 'tatonnement' adjustment process. [See, for example, some standard texts in international trade theory, such as Caves and Jones (1981) or Dixit and Norman (1980)].

More recently, the literature has focused on the three-country, two-commodity model, as it is felt that this framework leads to significantly different results, compared to the two-country counterpart. In particular, Brecher and Bhagwati (1981) and Yano (1983) have noted, in the context of a three agent, two commodity model that it is possible for the donor to be better off after the transfer, even under local stability of the Walrasian equilibrium. Gale (1974) has noted this phenomenon with a concrete algebraic example, and has established that the Walrasian equilibrium in his example is unique. (It turns out in this two-good example that the equilibrium is also locally stable as the reader can easily verify.) Gale's example involved L-shaped indifference curves for the three agents; an algebraic example with the same content as Gale's, but involving smooth preferences for the three agents was constructed by Aumann and Peleg (1974). This example, incidentally, shows a close relationship between the welfare effect of the transfer problem, and the welfare effect of the problem of 'immiserizing growth', which had been studied earlier quite extensively in international trade theory by Bhagwati (1958), and Johnson (1967). In a recent paper, Bhagwati, Brecher and Hatta (1983) have undertaken a thorough analysis of the three-country, two-commodity case integrating the transfer problem, the problem of immiserizing growth, and the theory of 'distortions'. [A rather complete history and bibliography of contributions to the transfer problem, immiserizing growth and distortions is contained in this paper, so we have deliberately avoided going into all of the details; see, however, remark (iii) following theorem 1.]

The purpose of our investigation is to provide a definitive result on the welfare effect of a transfer, in a general equilibrium exchange model, with no arbitrary restrictions on either the number of commodities traded or the number of countries. Furthermore, in contrast to the approach in the literature we would like to get 'global' results; that is, results in which the transfer is not restricted to be 'small'. The point that one should attempt to obtain 'global' comparative static results was emphasized, among others, by Morishima (1964, p. 3). This is analytically harder than the more prevalent 'local' analysis. We feel that Morishima's point is a serious one since an infinitesimally small change in a parameter is *not* what one has in mind when one formulates a typical comparative static proposition.

Our main result may be stated as follows: when there are an arbitrary number of countries and commodities, the donor country is worse off after the transfer, if (a) all goods are 'net substitutes' for the donor country; (b) all goods are 'gross substitutes' for the other countries; and (c) all goods are 'normal' for all countries.

One can see the relationship between this result and the set of necessary conditions for donor enrichment obtained by Bhagwati, Brecher and Hatta (1983), in the context of a two-good, three-country model involving bilateral transfers. (That is, in their model there is one donor country, another recipient country, and a third 'non-participant' country.) Let good x be the export commodity of the recipient country. Assuming that the relevant equilibria are locally stable, they note that if a transfer enriches the donor, then *either* x is an inferior good to the recipient *or* the offer curve of the non-participant country is inelastic (such that the export supply of x by the recipient country falls as the relative price of x rises).

Notice that our condition that all goods are normal for all countries rules out the first possibility of donor enrichment mentioned by Bhagwati, Brecher and Hatta. Also, the assumption that all goods are gross-substitutes for all countries except the donor country, implies in particular that the non-participant country's offer curve is elastic, ruling out thereby the second possibility of donor enrichment mentioned by Bhagwati, Brecher and Hatta. This makes it understandable why our sufficient conditions for 'no transfer paradox' work. Of course, one should bear in mind the differences between the exercise in this paper and that of Bhagwati, Brecher and Hatta. We consider a multilateral transfer in a pure-exchange world economy with an arbitrary (finite) number of commodities and countries. Their exercise is couched in terms of bilateral transfers in a two-commodity, three-country model in a world allowing exchange and production. [See also remark (ii) following theorem 1 below.]

2. The model

We consider a model of the world economy, in which there are n *commodities*, indexed $j=1,\ldots,n$, and m *countries*, indexed $i=1,\ldots,m$. The *total endowment* of the world economy is given, and denoted by e (where e is in R^n_{++}).

Each country has an endowment vector, the sum of all such endowment vectors being the total endowment of the world economy. In our notation, the endowment of the ith country is e^i in R^n_{++}, and

$$\sum_{i=1}^{m} e^i = e.$$

The *distribution of endowments* is then denoted by $E \equiv (e^1, e^2, \ldots, e^m)$.

The *preferences* of country i are represented by a continuous *utility function*, $u^i: R^n_+ \to R$. For each $i, D^i = \{x \text{ in } R^n_+ : u^i(x) > u^i(0)\}$. For each i, the following assumptions on u^i are maintained:

(A.1) (a) $x \geqq \bar{x} \geqq 0$ implies $u^i(x) \geqq u^i(\bar{x})$

(b) $x \gg \bar{x} \geqq 0$ implies $u^i(x) > u^i(\bar{x})$

(A.2) $\bar{x}$ in $D^i, x \neq \bar{x}, u^i(x) \geqq u^i(\bar{x})$ implies $u^i[\theta x + (1-\theta)\bar{x}] > u^i(\bar{x})$ for $0 < \theta < 1$.

An *allocation* is denoted by $X \equiv (x^1, \ldots, x^m)$ with x^i in R^n_+ for each i, and

$$\sum_{i=1}^{m} x^i \leqq e.$$

An *equilibrium* (given E) is a pair (X, p) such that:

$$X \text{ is an allocation,} \tag{1}$$

$$p \text{ is in } R^n_{++}, pe = 1, \tag{2}$$

$$\text{For each } i, x^i \text{ is in } B^i = \{x \text{ in } R^n_+ : px \leqq pe^i\} \text{ and } u^i(x^i) \geqq u^i(x) \text{ for all } x \text{ in } B^i. \tag{3}$$

A few remarks on the definition of equilibrium are in order. Given the assumptions on u^i, condition (3) implies $px^i = pe^i$, so

$$p\left[e - \sum_{i=1}^{m} x^i\right] = 0. \tag{4}$$

Furthermore, by condition (3), p is in R^n_{++}. Finally, since X is an allocation, and p is in R^n_{++}, so using (4), one gets:

$$\sum_{i=1}^{m} x^i = e. \tag{5}$$

We are interested in examining how an equilibrium changes when one country 'transfers' some of its endowment to another country (or to several of the other countries). This exercise clearly belongs to the more general class of comparative static problems concerned with the effect of a change in the distribution of endowments on the set of equilibrium allocations.

In describing 'transfers', we can suppose, without loss of generality, that country 1 *pays the transfer* (i.e. it is the 'donor' country), and some or all of the other countries ($i = 2, \ldots, m$) *receive the transfer*. Keeping this in mind, we can define a set

$$T=\{(E,\bar{E}):E \text{ and } \bar{E} \text{ are distributions of endowments,}$$

$$\bar{e}^1<e^1,\bar{e}^i\geqq e^i \text{ for } i=2,\ldots,m\}.$$

Thus, when we write $(E,\bar{E})$ is in T we mean that E is the distribution of endowments *before* the transfer, and $\bar{E}$ *after* the transfer.

Now, let $(E,\bar{E})$ be in T, (X,p) be an equilibrium given $E,(\bar{X},\bar{p})$ be an equilibrium given $\bar{E}$. The problem we are concerned with is the following. How do the post-transfer welfare levels $[u^1(\bar{x}),u^2(\bar{x}),\ldots,u^m(\bar{x})]$ compare to the pre-transfer welfare levels $[u^1(x),u^2(x),\ldots,u^m(x)]$?

3. Sufficient conditions for the normal welfare effect of a transfer

We wish to investigate the transfer problem in a many-country, many-commodity setting. Our purpose is to provide a set of sufficient conditions under which the 'normal' welfare effect of a transfer will occur (that is, the donor will be worse off after the transfer). It is well known that definitive results are hard to come by at this level of generality. Thus, we will need quite strong conditions on our model in order to obtain the result we are looking for. Specifically, we show that if all goods are net substitutes for the donor, and if all goods are gross substitutes for all the other countries, and all goods are normal for all countries, then the 'normal' welfare effect of a transfer will occur. It might be useful to provide an informal discussion to explain *why* these conditions produce the desired result.

Note that, under our assumptions, prices of all commodities are positive before and after the transfer, so supply equals demand in each market, in each equilibrium. If prices have not changed at all after a transfer, there is nothing to prove. The donor is obviously worse off.

If prices have changed, then some prices have gone up, and others have gone down. Consider a good whose price after transfer has fallen *the most* relative to its price before the transfer. Call this good k. Then note that the prices of all goods after the transfer, *relative to the price of good k after the transfer* must be at least as large as the prices of all goods before the transfer, *relative to the price of good k before the transfer*. This fact allows one, as in general equilibrium theory, to exploit the net-substitute and gross-substitute assumptions.

Focus now on the demand for good k. All countries except the donor have post-transfer endowments at least as large as the pre-transfer levels (with at least one country having a larger endowment in some good). Because all goods are normal and gross substitutes for each of them, the post-transfer demand for good k must be at least as large for all these countries, and strictly larger for at least one of them, than the pre-transfer level. If the donor is at least as well off after the transfer, then since all goods are normal and net substitutes for him, the post-transfer demand for good k must be at least as large for him as the pre-transfer level. Adding up, the world post-transfer demand for good k exceeds the world pre-transfer demand for good k. This is impossible since either magnitude equals the world supply of good k, which has not changed. Consequently, the donor must be worse off after the transfer.

We now proceed to provide a rigorous statement and proof of our theorem. For this purpose, let us start by recalling a few facts from the theory of demand. We take these facts (and others to be noted later) to be sufficiently well known that one need not go into the details of their derivations. [The reader can consult a standard text like Varian (1978).] For each $i=1,\dots m$ given p in R^n_{++} and d in R^n_{++}, there is a unique solution $g^i(p,d)$ to the following problem:

$$v^i(p,d) \equiv \max u^i(x)$$

s.t.

$$px \leqq pd, x \text{ in } R^n_+.$$

That is, for each $i=1,\dots,m$, there is an *ordinary* (*Marshallian*) *demand function*, $g^i(p,d)$, and an indirect utility function, $v^i(p,d)$ for p in R^n_{++}, d in R^n_{++}.

For our next fact, it is convenient to denote the range of u^i by Q^i.

Then, for each $i=1,\dots,m$, given p in R^n_{++}, w in Q^i, $w>u^i(0)$, there is a unique solutions $h^i(p,w)$ to the following problem:

$$M^i(p,w) \equiv \min px$$

s.t.

$$u^i(x) \geqq w, x \text{ in } R^n_+.$$

That is, for each $i=1,\dots,m$, there is a *compensated* (*Hicksian*) *demand function*, $h^i(p,w)$ and an *expenditure function* $M^i(p,w)$ for p in R^n_{++}, w in Q^i, $w>u^i(0)$.

In order to prove the main result of this section, we need the following additional assumptions. These assumptions are stated without assuming differentiability of the demand functions, following the style of Nikaido (1968, p. 305).

(A.3) For each i in $[2,\dots,m]$, and for $p,\bar{p}$ in R^n_{++}, d in R^n_{++}, $\bar{p} \geqq p$ implies

$$g^i_k(\bar{p},d) \geqq g^i_k(p,d) \text{ if } k \notin \{j \mid \bar{p}_j > p_j\}.$$

(A.4) For $p, \bar{p}$ in R^n_{++}, w in Q^1, $w > u^1(0), \bar{p} \geqq p$ implies

$$h^1_k(\bar{p}, w) \geqq h^1_k(p, w) \text{ if } k \notin \{j | \bar{p}_j > p_j\}.$$

(A.5) For each i in $[1, \ldots, m]$, for p in $R^n_{++}, d, \bar{d}$ in R^n_{++}, $\bar{d} > d$ implies

$$g^i_j(p, \bar{d}) > g^i_j(p, d) \text{ for } j = 1, \ldots, n.$$

Note that (A.3) says that all goods are 'gross substitutes' for countries $[2, \ldots, m]$, while (A.4) says that all goods are 'net substitutes' for country 1 (which, it will be recalled, is the 'donor' country). Finally, (A.5) says that all goods are 'normal' for all countries. [Assumptions (A.1)–(A.5) are consistent with arbitrary Cobb–Douglas utility functions for all countries.]

For p in R^n_{++}, and d in R^n_{++}, it is known that the indirect utility function, $v^1(p, d)$ is continuous in (p, d), by the Maximum Theorem [for a statement of the Maximum Theorem, see Berge (1963, p. 116)] and increasing in each component of d [by (A.2)]. Thus, given any p in R^n_{++}, w in Q^1, $w > u^1(0)$, there is d in R^n_{++}, such that $v^1(p, d) = w$. Similarly, for $\bar{w}$ in Q^1, $\bar{w} > w$, there is $\bar{d} > d$ such that $v^1(p, \bar{d}) = \bar{w}$. Now, $h^1[p, v^1(p, d)] = g^1[p, d]$ and $h^1[p, v^1(p, \bar{d})] = g^1[p, \bar{d}]$. Using (A.5), we therefore have $\bar{w} > w$ implying

$$h^1_j[p, \bar{w}] > h^1_j[p, w] \text{ for } j = 1, \ldots, n. \tag{6}$$

Theorem 1. Suppose assumptions (A.1)–(A.5) *hold. Let* $(X, p)[(\bar{X}, \bar{p})]$ *be an equilibrium given* $E[\bar{E}]$. *If* $(E, \bar{E})$ *is in* T, *then*

$$u^1(\bar{x}^1) < u^1(x^1). \tag{7}$$

Proof Since (X, p) is an equilibrium given E, so p is in R^n_{++}, x^i is in R^n_{++}, e^i is in R^n_{++}, and $u^i(x^i) > u^i(0)$ for i in $[1, \ldots, m]$. Similarly, $(\bar{X}, \bar{p})$ is an equilibrium given $\bar{E}$, so $\bar{p}$ is in R^n_{++}, $\bar{x}^1$ is in R^n_{++}, $\bar{e}^i$ is in R^n_{++}, and $u^i(\bar{x}^i) > u^i(0)$. Using the fact that (X, p) and $(\bar{X}, \bar{p})$ are equilibria given E and $\bar{E}$, respectively,

$$\sum_{i=1}^{m} g^i(p, e^i) = e = \sum_{i=1}^{m} g^i(\bar{p}, \bar{e}^i). \tag{8}$$

Let $w^1 = u^1(g^1(p, e^1))$, $\bar{w}^1 = u^1(g^1(\bar{p}, \bar{e}^1))$. Then, since $g^1(p, e^1) = h^1(p, w^1)$ and $g^1(\bar{p}, \bar{e}^1) = h^1(\bar{p}, \bar{w}^1)$, so (8) becomes:

$$h^1(p, w^1) + \sum_{i=2}^{m} g^i(p, e^i) = h^1(\bar{p}, \bar{w}^1) + \sum_{i=2}^{m} g^i(\bar{p}, \bar{e}^i). \tag{9}$$

Let $\min_j(\bar{p}_j/p_j)=(\bar{p}_k/p_k)$. Then defining $q=(p/p_k), \bar{q}=(\bar{p}/\bar{p}_k)$, we have

$$[h^1(q,w^1)-h^1(\bar{q},\bar{w}^1)]+\sum_{i=2}^{m}[g^i(q,e^i)-g^i(\bar{q},\bar{e}^i)]=0 \tag{10}$$

and

$$\bar{q}_j=\bar{p}_j/\bar{p}_k \geqq p_j/p_k=q_j \; j=1,\dots,n. \tag{11}$$

Suppose now, contrary to (7), that $\bar{w}^1 \geqq w^1$. Then, note that $h_k^1(q,w^1)-h_k^1(\bar{q},\bar{w}^1)=h_k^1(q,w^1)-h_k^1(\bar{q},w^1)+h_k^1(\bar{q},w^1)-h_k^1(\bar{q},\bar{w}^1)$. Now, using (11) and (A.4), $h^1(\bar{q},w^1)\geqq h_k^1(q,w^1)$. And, using (6) and $\bar{w}^1 \geqq w^1$, $h_k^1(\bar{q},\bar{w}^1)\geqq h_k^1(\bar{q},w^1)$. Hence,

$$h_k^1(q,w^1)-h_k^1(\bar{q},\bar{w}^1)\leqq 0. \tag{12}$$

Next, for $i=2,\dots,m$, we have $g_k^i(q,e^i)-g_k^i(\bar{q},\bar{e}^i)=g_k^i(q,e^i)-g_k^i(\bar{q},e^i)+g_k^i(\bar{q},e^i)-g_k^i(\bar{q},\bar{e}^i)$. Now, using (11) and (A.3), $g_k^i(\bar{q},e^i)\geqq g_k^i(q,e^i)$. And, using (A.5) and $\bar{e}^i \geqq e^i [i=2,\dots,m]$, we have $g_k^i(\bar{q},\bar{e}^i)\geqq g_k^i(\bar{q},e^i)$. Also, for some i in $[2,\dots,m]$, $\bar{e}^i > e^i$, so using (A.5), $g_k^i(\bar{q},\bar{e}^i)>g_k^i(\bar{q},e^i)$ for this i. Thus,

$$g_k^i(q,e^i)-g_k^i(\bar{q},\bar{e}^i)\leqq 0, \quad \text{for all} \quad i \text{ in } [2,\dots,m] \tag{13}$$

and

$$g_k^i(q,e^i)-g_k^i(\bar{q},\bar{e}^i)<0, \quad \text{for some } i \text{ in } [2,\dots,m]. \tag{14}$$

Using (13) and (14),

$$\sum_{i=2}^{m}[g_k^i(q,e^i)-g_k^i(\bar{q},\bar{e}^i)]<0. \tag{15}$$

But (12) together with (15) contradicts (10). Hence $\bar{w}_1 < w_1$, which establishes (7). Q.E.D.

4. Remarks

(i) Polterovich and Spivak (1983) have used the m-agent, n-commodity framework to obtain a result similar to ours. In the context of the case where there is one donor and one recipient, their result can be summarized as follows. If all goods are normal and gross substitutes for *all* countries, then the donor and recipient cannot be *both* better off after a transfer. Their assumptions are not strictly comparable to ours, nor is their result. They also look at the welfare of members of coalitions of arbitrary sizes, and they work with demand *correspondences* rather than *functions.*

(ii) We observed in section 1 how our result is related to the work of Bhagwati, Brecher and Hatta (1983). However, for our proof, it is clear that we need to assume normality for the donor country, while they do not need this assumption. This difference, we think is a consequence of their two-good framework together with the assumption of local stability equilibrium. In terms of their model, if the transfer enriches the donor, then the price of x (the export of the recipient) must fall. For this to happen under local stability, there must be a world excess supply of x *at the original price*, but at the new endowments. But, at the original price and new endowments, the non-participant's demand is the same as in the initial equilibrium, while the recipient's demand has risen (when x is normal for the recipient). Consequently, to produce the world excess supply, the demand for x by the donor at the original price and the new endowments must be lower than in the initial equilibrium. Since the donor's endowment is now smaller, x is a *normal* good, for this range. Our theorem 1 can then be invoked to contradict the original premise of donor enrichment. Thus, in Bhagwati, Brecher and Hatta's two-good case with local stability, our method of proof does not need the normality assumption for the donor, because normality at the relevant stage in our argument is already implied by the other assumptions.

(iii) We observed in section 1 that there is a close relationship between the transfer proboem and that of immiserizing growth. This is reinforced by Mantel's (1984) work on immiserizing growth, with arbitrary number of countries and commodities, which shows that if the set of three sufficient conditions we have used in section 3 are satisfied, then growth cannot be immiserizing. We are grateful to Avinash Dixit and Andreu Mas-Collel for pointing this out to us. [See also Hatta (1984).]

References

Aumann, R.J. and B. Peleg, 1974, A note of Gale's example, Journal of Mathematical Economics 1, 209–211.

Berge, C., 1963, Topological spaces (Macmillan, New York).

Bhagwati, J.N., 1958, Immiserizing growth: A geometrical note, Review of Economic Studies 25, 201–205.

Bhagwati, J.N., R.A. Brecher and T. Hatta, 1983, The generalized theory of transfers and welfare: Bilateral transfers in a multilateral world, American Economic Review 73, 606–618.

Brecher, R.A. and J.N. Bhagwati, 1981, Foreign ownership and the theory of trade and welfare, Journal of Political Economy 89, 497–511.

Caves, R. and R. Jones, 1981, World trade and Payments: An introduction (Little Brown, Boston, Third edition).

Dixit, A. and V. Norman, 1980, Theory of international trade (Cambridge University Press, Cambridge).

Gale, D., 1974, Exchange equilibrium and coalitions: An example, Journal of Mathematical Economics 1, 63–66.

Hatta, I., 1984, Immiserizing growth in a many-commodity setting, Journal of International Economics 17, 325–334.
Johnson, H.G., 1967, The possibility of income losses from increased efficiency or factor accumulation in the presence of tariffs, Economic Journal 77, 151–154.
Mantel, R., 1984, Substitutability and the welfare effects of endowment increases, Journal of International Economics 17, 325–334.
Morishima, M., 1964, Equilibrium, stability and growth (Oxford University Press, Oxford).
Nikaido, H., 1968, Convex structures and economic theory (Academic Press, New York).
Polterovich, V.M. and V.A. Spivak, 1983, Gross substitutability of point-to-set correspondences, Journal of Mathematical Economics 11, 117–140.
Samuelson, P.A., 1952, The transfer problem and transport costs: The terms of trade when impediments are absent, Economic Journal 62, 278–304.
Samuelson, P.A. 1954, The transfer problem and transport costs, II: Analysis of effects of trade impediments, Economic Journal 64, 264–289.
Varian, H.R., 1978, Microeconomic analysis (W.W. Norton, New York).
Yano, M., 1983, Welfare aspects of the transfer problem, Journal of International Economics 15, 277–289.